

Age and Coin

1. Thomas purchases a selection of wrenches for his shop. His bill is \$78. He buys the same number of \$1.50 and \$2.50 wrenches, and half that many of \$4 wrenches. The number of \$3 wrenches is one more than the number of \$4 wrenches. How many of each did he purchase?

1.5	2x	Q+y
2.5	2x	10
4	x	5
3	x+1	6

$$2x(1.5) + 2x(2.5) + 4x + 3(x+1) = 78$$

$$3x + 5x + 4x + 3x + 3 = 78$$

$$15x = 75$$

$$x = 5$$

2. 8 years ago, Jimmy was three times as old as Anne. In 4 years, Jimmy will be 3/2 of Anne's age. How old will Anne be in 2 years?

- a) 8
b) 12
c) 14
d) 20
e) 22

Method 2 - use one variable

	8 years ago	In 4 years
Jimmy	3x	3x + 12
Anne	x	x + 12

$$3x + 12 = \frac{3}{2}(x + 12)$$

$$6x + 24 = 3x + 36$$

$$3x = 12$$

$$x = 4$$

$$x + 10 = 14$$

Method 1

$$J - 8 = 3(A - 8)$$

$$J + 4 = \frac{3}{2}(A + 4)$$

$$A + 2 = ?$$

3. John is 6 years younger than Tina. Beth is 3 times as old as Steve. Tina is twice as old as Mike. Steve is 2 years older than Mike. If the sum of everyone's age in 2 years is 102, how old is John?

- a) 10
b) 12
c) 14
d) 20
e) 36

John	2x - 6	+2
Tina	2x	+2
Beth	3x + 6	+2
Steve	x + 2	+2
Mike	x	+2

$$2(10) - 6 = 14$$

$$9x + 12 = 102$$

$$9x = 90$$

$$x = 10$$

$$x = 10$$

Steve is twice as old as Sylvie. Sylvie is three years older than Jacob. 4 years ago, Sylvie was twice Jacob's age. How old is Steve?

- a) 20
- b) 14
- c) 10
- d) 7
- e) 5

	Now	4 years ago
STEVE	$2x$	$2x - 4$
SYLVIE	x	$x - 4$
JACOB	$x - 3$	$x - 7$

$$x - 4 = 2(x - 7)$$

$$x - 4 = 2x - 14$$

$$10 = x$$

$$2x = 20$$

Algebra

4. Find the solution set for the following inequality $-2|3-2x| < 14$?

$$|3-2x| > -7 \quad \leftarrow$$

$$\begin{array}{ll} 3-2x > -7 & \text{or } -(3-2x) > -7 \\ -2x > -10 & 3-2x < 7 \\ x < 5 & -2x < 4 \\ -2 < x < 5 & x > -2 \end{array}$$

Important to note $|any| \geq 0$ always; thus x in this case can be anything because $|3-2x| > \text{Negative}$ always

Reality says $x = \text{anything}$

- any time you work with absolute values you should always go back and check!!!

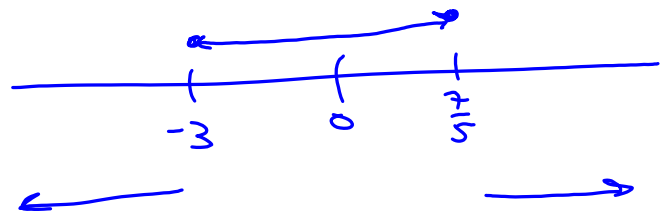
5. Find the solution set for the following inequality $|3x-2| \leq |2x-5|$?

$$\begin{aligned} (3x-2)^2 &\leq (2x-5)^2 \\ 9x^2 - 12x + 4 &\leq 4x^2 - 20x + 25 \\ 5x^2 + 8x - 21 &\leq 0 \\ (5x-7)(x+3) &\leq 0 \end{aligned}$$

$$\begin{aligned} 5x &= 7 & x &= -3 \\ x &= \frac{7}{5} \end{aligned}$$

Test $x = 0$? ; OK

$$-3 \leq x \leq \frac{7}{5}$$



if 0 test did not work...

6. Is $|x-z| + |x| = |z|$?

(1) $zy < xy < 0$

(2) $y > 0$

$$\overset{\text{Left}}{|x-z|} = \overset{\text{Right}}{|z| - |x|} ?$$

- (1) if $y > 0$ then $z > x > 0$
+ + ; yes

if $y < 0$ then $z < x < 0$
- - ; yes

x	z	
3	5	$z = z$
6	7	$1 = 1$
-3	-5	$z = z$
-6	-7	$1 = 1$

$|a-b|$ = pos change/dif
 $|a| - |b|$ = dif ignore the signs
however result could be
negative if $|b| > |a|$

Is $|x-y| > |x| - |y|$?

(1) $y < x$

(2) $xy < 0$

~~| x | y |
|---|---|
| + | + |
| + | - |~~

Left Right
= ; No
> ; yes

(3)

-(2)

+ -

Left vs Right
> ; yes

7. If $s - \frac{1}{s} < \frac{1}{t} - t$ is $s < t$?

(1) $s > 1$

(2) $t > 0$

-(1)

$S > 1$

Here my first thought
is if $s > 1$ can T also
be > 1 ? **No it can't**

Thus $T < 1$

So is $s < T$? **No!**

(A)

~~(2)~~

s	T
$\frac{1}{2}$	$\frac{1}{3}$
$\frac{1}{3}$	$\frac{1}{2}$

• No

; yes

Fact

$$S + T < \frac{1}{s} + \frac{1}{T}$$

How could this be possible?

fractions?

Negatives?

8. If $a \neq -b$, is $\frac{a-b}{a+b} < 1$?

(1) $b^2 > a^2$

(2) $a - b > 1$

-(1) $0 > a^2 - b^2$

$0 > (a-b)(a+b)$
 $(+) \cdot (-)$; yes

(A)

$(-) \cdot (+)$; yes ~~no~~

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x-y)^2 = x^2 - 2xy + y^2$$

$$(x-y)(x+y) = x^2 - y^2$$

a	b	
5	2	$\frac{3}{7}$; yes
5	-2	$\frac{7}{3}$; No

$$40x > 12y$$

$$10x > 3y$$

9. Is $\frac{x}{12} > \frac{y}{40}$? $y < \frac{10}{3}x$?

(1) $10x < 3y - 6$

(2) $12x - 7 > 4y$

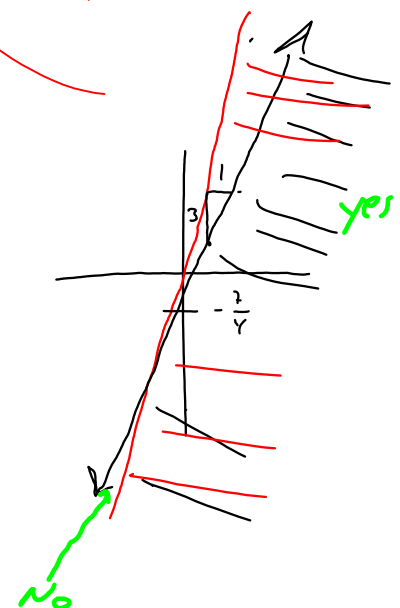
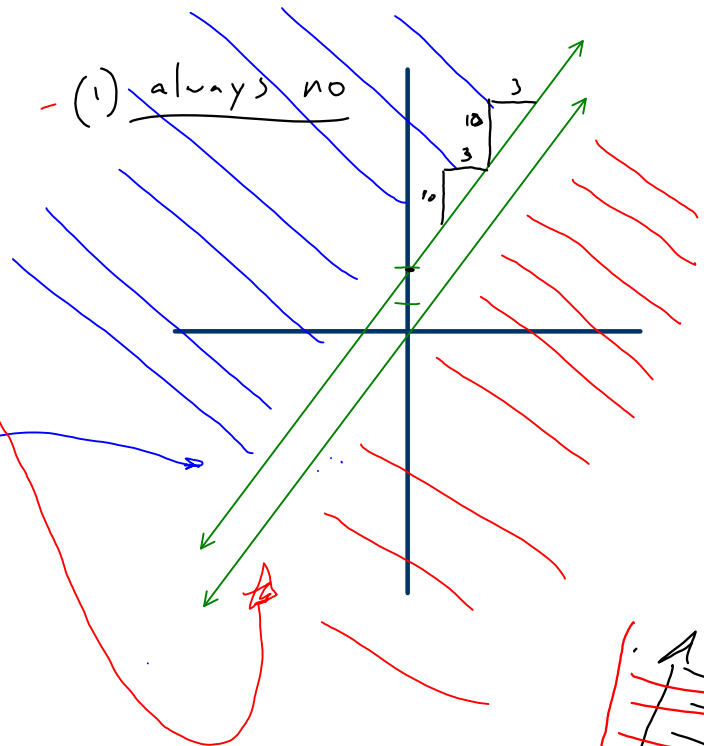
(1) $3y > 10x + 6$
 $y > \frac{10}{3}x + 2$

~~(2)~~ $4y < 12x - 7$
 $y < 3x - \frac{7}{4}$
 $y < \frac{10}{3}x$?

A

since the slopes
are different the
lines will cross
so sometimes
yes sometimes no

(1) always no



Averages

Think about the sum $\Sigma = \bar{x} \cdot n$

10. The average weight of a class of 44 students is 36 kg. If the weight of the teacher is included, then the average weight of the class increases by 1 kg. What is the weight of the teacher?

$$44 \cdot 36 + T = 45 \cdot 37$$

$$T = 45 \cdot 37 - 44 \cdot 36$$

$$\begin{array}{r} 3 \\ 145 \\ 37 \\ \hline 315 \\ 1356 \\ \hline 1665 \end{array} \quad \begin{array}{r} 21 \\ 44 \\ 36 \\ \hline 204 \\ 1320 \\ \hline 1584 \end{array}$$

$$\begin{array}{r} 1665 \\ - 1584 \\ \hline 81 \end{array}$$

$$\boxed{T = 81}$$

11. In Tom's favorite game he has a certain average for the first 25 rounds. After scoring 140 points in the 26th his average improves by 3 points. What is his new average after the 26th round?

old $\bar{x} = x$

new $\bar{x} = x + 3$

Σ of first 25

$$25x$$

Σ of 26

$$26(x+3)$$

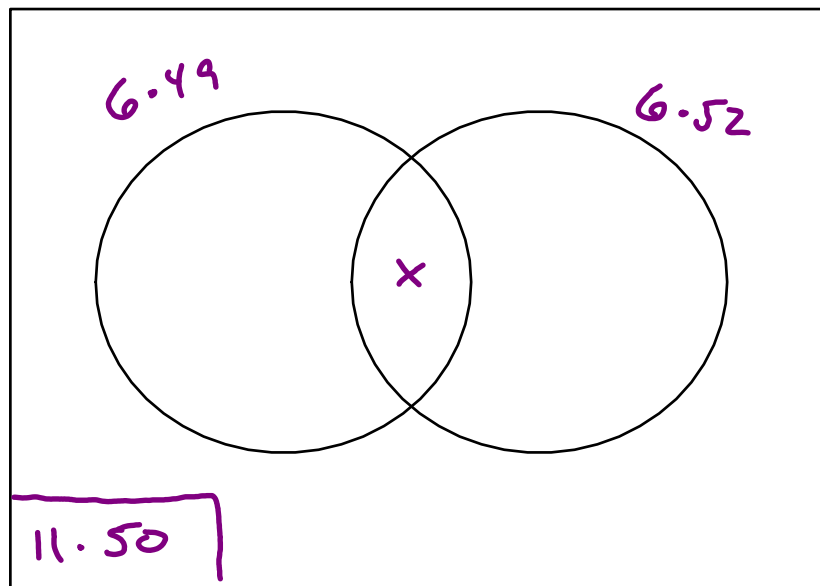
$$25x + 140 = 26(x+3)$$

$$140 = x + 78$$

$$62 = x$$

$$65 = x + 3$$

12. The average of 11 games is 50. The average of the first 6 is 49 and the last 6 is 52. What is the result of the 6th game?



$$\begin{aligned} 17 \cdot 62 - 1662 \\ 162 \\ \textcircled{62} \\ 12 / (17 - 16) \end{aligned}$$

$$\begin{aligned} 11 \cdot 50 &= 6 \cdot 49 + 6 \cdot 52 - x \\ 550 &= 6 \cdot 101 - x \\ x &= 606 - 550 \\ x &= 56 \end{aligned}$$

13. Committee X and Committee Y, which have no common members, will combine to form Committee Z. Does Committee X have more members than Committee Y?

- (1) The average (arithmetic mean) age of the members of Committee X is 25.7 years and the average age of the members of Committee Y is 29.3 years.
(2) The average (arithmetic mean) age of the members of Committee Z will be 26.6 years.

$$\begin{aligned} \cancel{\Sigma} \text{ of ages } X &\rightarrow 25.7(X) \\ \Sigma \text{ of ages } Y &\rightarrow 29.3(Y) \end{aligned}$$

$$\cancel{\Sigma} \text{ of ages } (X+Y) \rightarrow 26.6(X+Y)$$

$$-\left(\frac{1}{2}\right) 25.7(X) + 29.3(Y) = 26.6(X+Y)$$

$$27Y = 9X$$

$$3Y = X$$

$$Y : X$$

$$1 : 3$$

Let X = Qty of members from X

Let Y = Qty of members from Y

14. A farmer has 10 sheep 4 black and 6 white. He would like to take 3 of the sheep to get shaved. How many different groups of 3 sheep can he select that would have at least one black one?

Direct method

$$\begin{matrix} B & W & W & \text{or} & B & B & W & \text{or} & B & B & B \\ \binom{4}{1} & \binom{6}{2} & + & \binom{4}{2} & \binom{6}{1} & + & \binom{4}{3} \end{matrix}$$

$$\frac{4}{1} \cdot \frac{6 \cdot 5}{2!} + \frac{4 \cdot 3}{2!} \cdot \frac{6}{1} + \frac{4 \cdot 3 \cdot 2}{2!}$$

$$60 + 36 + 4 = 100$$

Indirect method

any 3 - www

$$\binom{10}{3} - \binom{6}{3}$$

$$\frac{10 \cdot 9 \cdot 8}{3!} - \frac{6 \cdot 5 \cdot 4}{3!}$$

$$120 - 20 = 100$$

15. There are 10 books on a book shelf, 5 Math, 3 History and 2 Art. If 2 books are selected at random how many different pairs of 2 different kinds of books can be selected?

Indirect

any 2 - HH AA MM

$$\binom{10}{2} - \binom{3}{2} - \binom{2}{2} - \binom{5}{2}$$

$$\frac{10 \cdot 9}{2!} - \frac{3 \cdot 2}{2!} - \frac{2 \cdot 1}{2!} - \frac{5 \cdot 4}{2!}$$

$$45 - 3 - 1 - 10$$

$$= 31$$

Direct

$\begin{matrix} \text{MH} \\ \binom{5}{1} \binom{3}{1} \end{matrix} + \begin{matrix} \text{MA} \\ \binom{5}{1} \binom{2}{1} \end{matrix} + \begin{matrix} \text{HA} \\ \binom{3}{1} \binom{2}{1} \end{matrix}$

$$5 \cdot 3 + 5 \cdot 2 + 3 \cdot 2$$

$$15 + 10 + 6 = 31$$

16. Four different science books, six different art books and two different novels are to be arranged on a shelf. How many different arrangements are possible if (a) all the books of one type must stand together, and (b) only the science books must stand together?

(a) $\boxed{SSSS} \quad \boxed{AAAAAA} \quad \boxed{NN}$
 $4! \cdot 6! \cdot 2! \cdot 3!$

(b) $\boxed{SSSS} \quad \boxed{A} \quad \boxed{A} \quad \boxed{A} \quad \boxed{A} \quad \boxed{A} \quad \boxed{A} \quad \boxed{N} \quad \boxed{N}$
 $9! \cdot 4!$

17. How many 3 digit numbers exist that have only one 7?

$$\begin{array}{ccc}
 \begin{array}{c} 7 \\ \hline 1 \end{array} & \begin{array}{c} N \\ \hline 0 \rightarrow 9 \\ \text{not } 7 \end{array} & \begin{array}{c} N \\ \hline 0-9 \\ \text{not } 7 \end{array} \rightarrow 1 \cdot 9 \cdot 9 = 81 \\
 \begin{array}{c} N \\ \hline 1-9 \\ \text{not } 7 \end{array} & \begin{array}{c} 7 \\ \hline \end{array} & \begin{array}{c} N \\ \hline 0-9 \\ \text{not } 7 \end{array} \rightarrow 8 \cdot 1 \cdot 9 = 72 \\
 \begin{array}{c} N \\ \hline \end{array} & \begin{array}{c} N \\ \hline \end{array} & \begin{array}{c} 7 \\ \hline \end{array} \rightarrow 8 \cdot 9 \cdot 1 = 72
 \end{array}$$

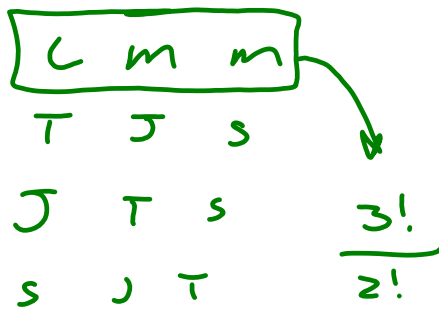
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18. A manager needs to hire 3 people to work in her marketing department, and one of which is to be the team leader. If there are 10 potential candidates, how many distinct ways can she fill these three positions?

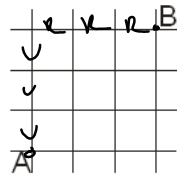
any 3

$$\binom{10}{3} \rightarrow \frac{10 \cdot 9 \cdot 8}{\cancel{3!}} \cdot \frac{\cancel{3!}}{2!}$$

$$10 \cdot 9 \cdot 8 = \underline{360}$$



19. If Roger lives at point A and works at point B, how many different ways can he travel to work if he must always take the shortest path?



U U U R R R

$$\frac{6!}{3!3!}$$

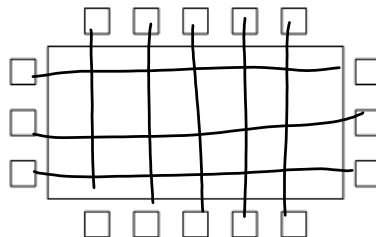
$$\frac{6 \cdot 5 \cdot 4}{2 \cdot 1} = 20$$

BAD 3!

TOO $\frac{3!}{2!}$

MISSION $\frac{7!}{2!2!}$

20. How many ways can Mike and John sit down at the table so that they don't sit directly across from each other?



$$\binom{16}{2} - 8 \cdot 2!$$

$$16 \cdot 15 - 8 \cdot 2!$$

$$240 - 16 = \boxed{224}$$

A firm is divided into four departments, each of which contains four people. If a project is to be assigned to a team of three people, none of which can be from the same department, what is the greatest number of distinct teams to which the project could be assigned?

- (A) 4^3
- (B) 4^4
- (C) 4^5
- (D) $6(4^4)$
- (E) $4(3^6)$

A	B	C	D
$\begin{pmatrix} 4 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 4 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 4 \\ 1 \end{pmatrix}$	
$4 \cdot 4 \cdot 4 \cdot \begin{pmatrix} 4 \\ 3 \end{pmatrix}$			
$4 \cdot 4 \cdot 4 \cdot \frac{4 \cdot 3 \cdot 2}{3!} = 4^4$			

WHAT IF THERE WERE 8 DEPARTMENTS
AND 5 people in each department?

$$\begin{pmatrix} 5 \\ 1 \end{pmatrix} \begin{pmatrix} 5 \\ 1 \end{pmatrix} \begin{pmatrix} 5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ 3 \end{pmatrix}$$

$$5 \cdot 5 \cdot 5 \cdot \frac{8 \cdot 7 \cdot 6}{3!}$$

$56 \cdot 5^3$

A committee of three people is to be chosen from the president and vice president of four different companies. What is the number of different committees that can be chosen if two people who work for the same company cannot both serve on the committee?

- (A) 16
- (B) 24
- (C) 28
- (D) 32
- (E) 40

A B C D
~~PV~~ ~~PV~~ PV PV

$$\frac{8 \cdot 6 \cdot 4}{3!}$$

$$\binom{2}{1} \binom{2}{1} \binom{2}{1} \binom{4}{3}$$

$$\frac{2 \cdot 2 \cdot 2 \cdot 4 \cdot 3 \cdot 2}{3!}$$

$$2^3 \cdot 4 = 32$$

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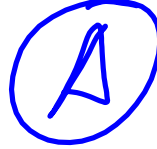
Coordinate Geometry

21. What is the value of k ?

(1) In the xy -coordinate system, (a, b) and $(a+3, b+k)$ are two points that lie on the line defined by the equation $x = 3y - 7$.

(2) $k^2 = 1$

~~(2)~~ $k = \pm 1$



-(1) with two points we can get the slope
from the equation of the line we can also
get the slope.

$$\frac{\Delta y}{\Delta x} = \frac{b+k-b}{a+3-a} = \frac{k}{3}$$

\swarrow slope \nearrow

$$3y = x + 7$$

$$y = \frac{1}{3}(x) + \frac{7}{3}$$

$$\frac{k}{3} = \frac{1}{3}$$

$$\boxed{k = 1}$$

22. What is the least possible distance between a point on the circle $x^2 + y^2 = 1$ and a point on the line $y = \frac{3}{4}x - 3$?

A) 1.4

B) $\sqrt{2}$

C) 1.7

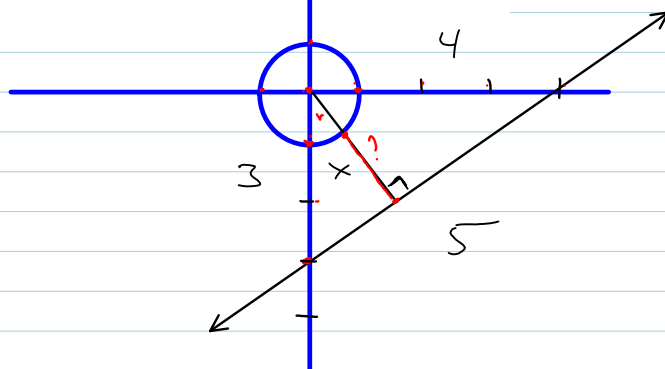
D) $\sqrt{3}$

E) 2

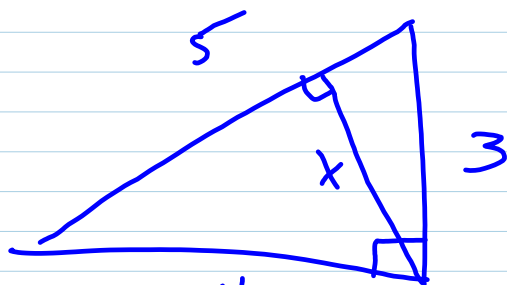
$$y = \frac{3}{4}x - 3$$

x	y
1	0
0	1
0	-1
-1	0

$$r = 1$$



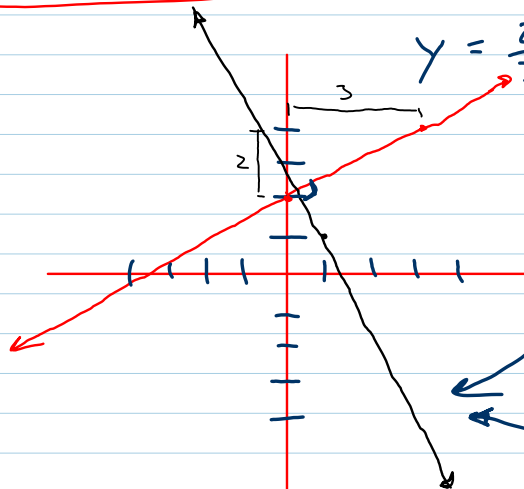
$$2.5 - 1 = 1.4$$



$$\frac{1}{2}(3 \cdot 4) = \frac{1}{2}5x$$

$$12 = 5x$$

$$2.4 = \frac{12}{5} = x$$



$$y = \frac{2}{3}x + 2$$

perpendicular (1,1)

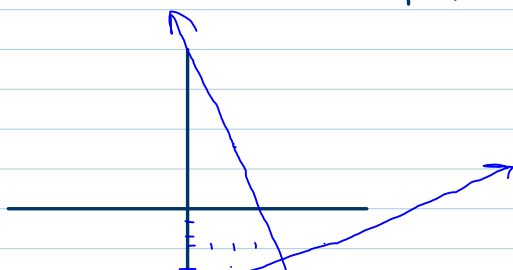
$$y = -\frac{3}{2}x + b_2 \quad (1,1)$$

$$1 = -\frac{3}{2} + b_2$$

$$\frac{5}{2} = b_2 \quad y = -\frac{3}{2}x + \frac{5}{2}$$

$$y = \frac{1}{3}(x) - 4$$

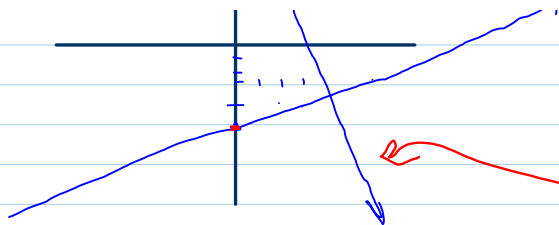
equation of \perp crosses at (2,3)
perpendicular



$$y = -3x + b_2$$

$$3 = -3(2) + b_2$$

$$9 = b_2$$



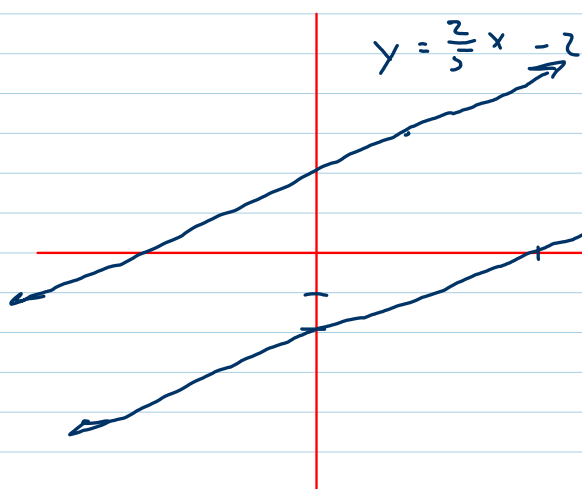
$$3 = -3(2) + b_2$$

$$9 = b_2$$

$$y = -3x + 9$$

$$5y = 2x - 10$$

what is the equation of the parallel line that crosses a $(2, 3)$



$$y = \frac{2}{5}x + b_2$$

$$3 = \frac{4}{5} + b_2$$

$$2\frac{1}{5} = b_2$$

$$y = \frac{2}{5}x + \frac{11}{5}$$

parallel lines have the same slope

23. In the xy plane line m has the equation $4x + y = k$. Line n passes through the origin and is perpendicular to line m. If point p has the coordinates $(r, r+1)$ and is on both line n and m what is the value of r?

- a) $-4/3$
- b) $1/4$
- c) -4
- d) $3/4$
- e) $4/3$

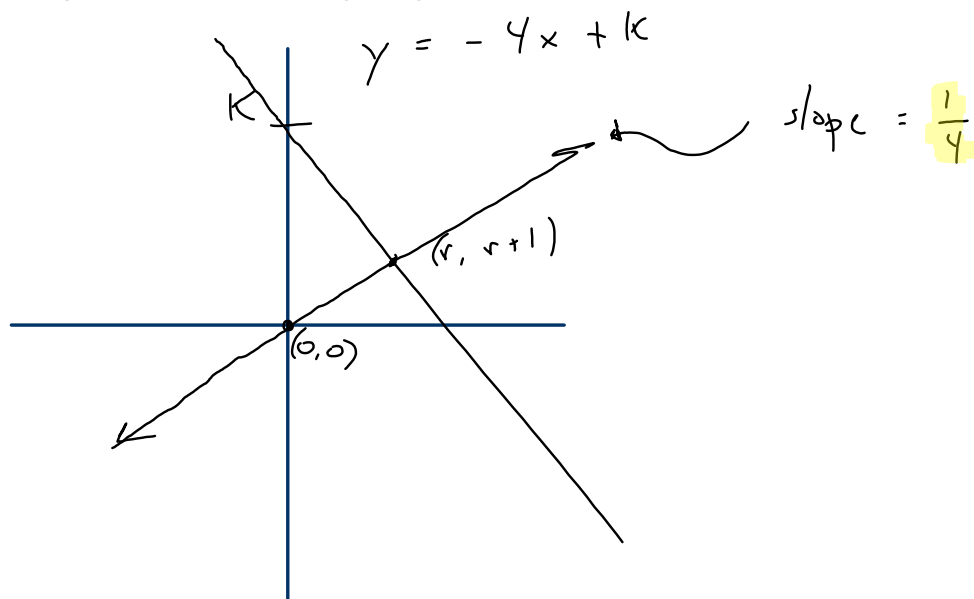
$$\frac{\Delta y}{\Delta x} = \frac{r+1}{r}$$

$$\frac{r+1}{r} = \frac{1}{4}$$

$$4r + 4 = r$$

$$3r = -4$$

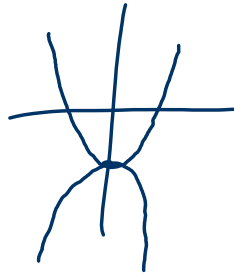
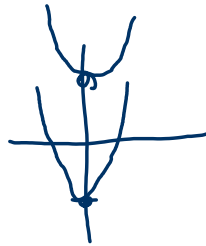
$$r = -\frac{4}{3}$$



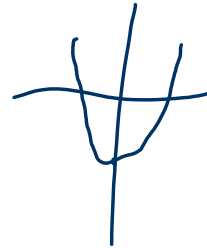
24. In xy -plane, $y=ax^2+bx+c$, does the graph intersect with x -axis?

(1) $a > 0$

(2) $c < 0$



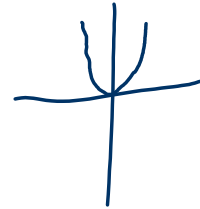
$-(1/2)$



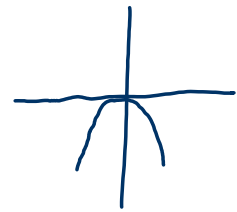
C

$$y = x^2$$

x	y
0	0
-1	1
1	1
-2	4
2	4



$$y = -x^2$$



25. If line k in the xy plane has equation $y = mx + b$, where m and b are constants, what is the slope of k ?

(1) k is parallel to the line with equation $y = (1-m)x + b + 1$

(2) k intersects the line with equation $y = 2x + 3$ at the point $(2, 7)$

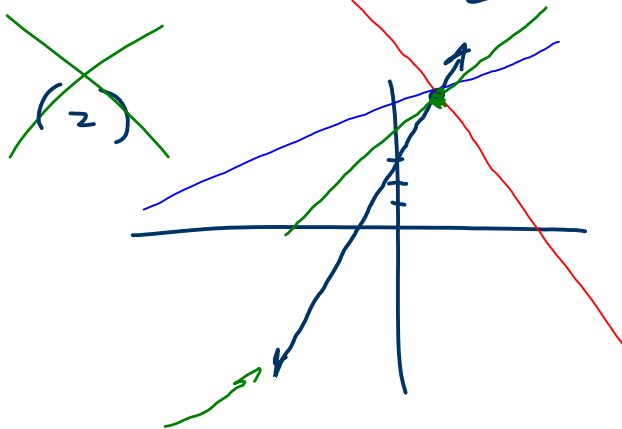
parallel lines have equal slopes

-(1)

$$m = 1 - m$$

$$2m = 1$$

$$m = \frac{1}{2} = \text{slope of } k$$



A

26. In the xy-plane, the line with equation $ax + by + c = 0$, where $abc \neq 0$, has slope $2/3$.
What is the value of b ?

(1) $a = 4$

(2) $c = -6$

$$by = -ax - c$$

$$y = -\frac{a}{b}x - \frac{c}{b}$$

$$-\frac{a}{b} = \frac{2}{3}$$

-(1) $a = 4$

~~(2)~~ $c = -6$

A

27. Simplify the following fractions:

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a) $\frac{2}{4+\sqrt{14}}$

$$\frac{2}{4+\sqrt{14}} \cdot \frac{4-\sqrt{14}}{4-\sqrt{14}} = \frac{2(4-\sqrt{14})}{16-14}$$

$$= 4 - \sqrt{14}$$

b) $\frac{2^{21}}{4^4(4^5-4^3)}$

$$\frac{2^{21}}{4^9 - 4^7} = \frac{2^{21}}{4^7(4^2-1)} = \frac{2^{21}}{(2^2)^7 \cdot 15}$$

$$= \frac{2^{21}}{2^{14} \cdot 15} = \frac{2^7}{15}$$

c) $\frac{3 \times 2^n - 4 \times 2^{n-2}}{2^n - 2^{n-1}}$

$$\frac{3 \cdot 2^n - 2^2 \cdot 2^n \cdot 2^{-2}}{2^n - 2^{-1} \cdot 2^n} = \frac{3 \cdot 2^n - 2^n}{2^n - \frac{1}{2} \cdot 2^n}$$

$$= \frac{2^n(3-1)}{2^n(1-\frac{1}{2})} = \frac{2}{\frac{1}{2}} = 4$$

28. Simplify the following roots:

a) $\sqrt{486}$

$$\sqrt{\boxed{33} \boxed{33} 32}$$

$$3 \cdot 3 \sqrt{3 \cdot 2} = 9\sqrt{6}$$

$$\begin{array}{c} 486 \\ \wedge \\ 9 \quad 54 \\ \wedge \quad \wedge \\ 3 \quad 3 \quad 3 \cdot 3 \cdot 3 \cdot 2 \end{array}$$

$$\begin{array}{r} 54 \\ 1 \overline{) 486} \\ \underline{45} \\ 36 \end{array}$$

b) $\sqrt{64 + 2^6}$

$$\sqrt{64 + 64} = \sqrt{2 \cdot 64}$$

$$= 8\sqrt{2}$$

c) $\sqrt[3]{100} \times \sqrt[6]{100}$

$$100^{\frac{1}{3}} \cdot 100^{\frac{1}{6}}$$

$$\frac{1}{3} + \frac{1}{6} = \frac{6+3}{18}$$

$$= 100^{\frac{1}{2}} = (10^2)^{\frac{1}{2}}$$

$$= \frac{1}{2}$$

$$= 10$$

29. Solve for n for the following equations:

a) $9^7 + 9^7 + 9^7 = 27^{n+1}$

$9^7 + 9^7 + 9^7 = (3^2)^{n+1}$
 $(3^2)^7 + (3^2)^7 + (3^2)^7 = 3^{2n+2}$
 $3^{14} + 3^{14} + 3^{14} = 3^{2n+2}$ ok
 $14 + 14 + 14 = 2n + 2$ wrong

$9^7(1+1+1) = (3^3)^{n+1}$
 $(3^2)^7 \cdot 3 = 3^{3n+3}$
 $3^{14} \cdot 3 = 3^{3n+3}$
 $3^{15} = 3^{3n+3}$

$15 = 3n + 3$

$5 = n + 1$

$4 = n$

b) $16^{n-9} + 4^{2n-18} = 256^{n-4}$

$(2^4)^{n-9} + (2^2)^{2n-18} = (2^8)^{n-4}$

$2^{4n-36} + 2^{4n-36} = 2^{8n-32}$

$2 \cdot 2^{4n-36} = 2^{8n-32}$

$2^{4n-35} = 2^{8n-32}$

$4n - 35 = 8n - 32$
 $-3 = 4n$

$-\frac{3}{4} = n$

c) $\left(\frac{1}{7}\right)^n \times \left(\frac{1}{9}\right)^{17} = \frac{1}{3^3 \times 21^{31}}$

Left
 $\boxed{7^n} \boxed{3^{34}}$

or

Right
 $\boxed{3^3} \boxed{3^{31}} \boxed{7^{31}}$

$$\boxed{n = 31}$$

in order for the left to equal the right we need the same amount of prime factors on each side.

Since there are 31 7's on the Right we will need 31 on the Left.

d) $\sqrt{(10n - 25)^3} = n^3$

$$\left[(10n - 25)^{\frac{3}{2}} \right]^{\frac{2}{3}} = \left[n^3 \right]^{\frac{2}{3}}$$

$$10n - 25 = n^2$$

$$n^2 - 10n + 25 = 0$$

$$(n - 5)(n - 5) = 0$$

$$\boxed{n = 5}$$

e) $2^n - 2^{n-2} = 3 \times 2^{13}$

$$2^n - 2^n \cdot 2^{-2} = 3 \cdot 2^{13}$$

$$2^n \left(1 - \frac{1}{4} \right) =$$

$$\frac{3}{4} \cdot 2^n =$$

$$2^n = \frac{4}{3} \cdot 3 \cdot 2^{13}$$

$$2^n = 2^2 \cdot 2^{13} = 2^{15}$$

$5^x - 5^{x-3} = 124 \cdot 5^y$, what is y in terms of x?

- a) x 3
- b) x - 6 -3
- c) x - 3 3
- d) 2x + 3 4
- e) 2x + 6 12

$$x = 3$$

insert ↗

$$5^3 - 5^0 = 124 \cdot 5^y$$

$$y = ?$$

golden rule
when you see variables
in the answers you
can insert

in this case simply

$$5^3 - 5^0 = 124 \cdot 5^Y$$

$$124 = 124 \cdot 5^Y$$

$$1 = 5^Y$$

$$0 = Y$$

1 can .
 in the case simply
 insert for x and
 then solve for y...

$5^x - 5^{x-3} = 124 \cdot 5^y$, what is y in terms of x ?

- a) x
- b) $x - 6$
- c) $x - 3$
- d) $2x + 3$
- e) $2x + 6$

$$5^x - 5^x \cdot 5^{-3} = 124 \cdot 5^y$$
$$5^x \left(1 - \frac{1}{125} \right) =$$

$$\frac{124}{125} \cdot 5^x =$$

$$5^x = \frac{125}{124} \cdot 124 \cdot 5^y$$

$$5^x = 5^3 \cdot 5^y$$

$$x = 3 + y$$

$$x - 3 = y$$

If $10^k = \frac{1}{2}$, then what is the value of 10^{k+3} ?

- a) 1000
- b) 500
- c) 50
- d) $\frac{1}{8}$
- e) $\frac{1}{16}$

$$10^{k+3} = 10^k \cdot 10^3$$

$$10^k = \frac{1}{2}$$

$$\frac{1}{2} \cdot 10^3 = 500$$

Functions

30. For all integers b , $\sim b \sim = b^2 - 2$. What is the value of $\sim(\sim 5 \sim) \sim$?

$$\sim b \sim = b^2 - 2$$

$$\sim 5 \sim = 25 - 2 = 23$$

$$\sim 23 \sim = 23^2 - 2 = 529 - 2 = 527$$

$$f(x) = 3x + 2$$

$$f(3) = 9 + 2 = 11$$

$$f(k) = 3k + 2$$

$$\begin{aligned} f(k-6) &= 3(k-6) + 2 \\ &= 3k - 18 + 2 \\ &= 3k - 16 \end{aligned}$$

$$f(z+3) = 14 \quad z = ?$$

$$\begin{aligned} f(z+3) &= 3(z+3) + 2 = 14 \\ &= 3z + 9 + 2 = 14 \\ &= 3z + 11 = 14 \\ &= 3z = 3 \\ &z = 1 \end{aligned}$$

$$f(x, y) = x^2 - 2y$$

$$\begin{aligned} f(4, b-1) &= 16 - 2(b-1) \\ &= 16 - 2b + 2 \\ &= 18 - 2b \end{aligned}$$

31. For numbers j and r , $r @ j = r \times (r - j)$. What is the value of x if $x @ 10 = -25$?

$$\begin{aligned} r @ j &= r(r - j) \\ x @ 10 &= x(x - 10) = -25 \\ &= x^2 - 10x + 25 = 0 \\ &= (x - 5)(x - 5) = 0 \\ &= \underline{x_1 = 5} \end{aligned}$$

32. For all integers x , $*x = x^2 - 6$. What are all possible values of b if $*(b-1)=10$?

$$\begin{aligned} *(b-1) &= (b-1)^2 - 6 = 10 \\ (b-1)^2 &= 16 \\ b-1 &= \pm 4 \\ b-1 &= 4 \quad \quad b-1 = -4 \\ \underline{b = 5} \quad \quad \underline{b = -3} \end{aligned}$$

33. If $f(x) = 125/x^3$, what is the value of $(f(5x)) \times (f(x/5))$ in terms of $f(x)$?

- a) $(f(x))^2$
- b) $f(x^2)$
- c) $(f(x))^3$
- d) $f(x^3)$
- e) 1

$$\begin{aligned} f(x) &= \frac{125}{x^3} \\ f(5x) &= \frac{125}{(5x)^3} = \frac{5^3}{5^3 \cdot x^3} = \frac{1}{x^3} \\ f\left(\frac{x}{5}\right) &= \frac{125}{\frac{x^3}{5^3}} = \frac{5^3 \cdot 5^3}{x^3} \\ \frac{1}{x^3} \cdot \frac{5^3 \cdot 5^3}{x^3} &= \left(\frac{5^3}{x^3}\right)^2 \end{aligned}$$

34. For which of the following functions f is $f(x) = f(1-x)$ for all x ?

a) $f(x) = 1-x$

b) $f(x) = 1-x^2$

c) $f(x) = x^2 - (1-x)^2$

d) $f(x) = x^2(1-x)^2$

e) $f(x) = x/(1-x)$

$x \stackrel{?}{=} 1$

$f(1) = f(0) ?$

~~a)~~ $f(1) = 0 \neq f(0) = 1$

~~b)~~ $f(1) = 0 \neq f(0) = 1$

~~c)~~ $f(1) = 1 \neq f(0) = -1$

\sim (d) $f(1) = 0 = f(0) = 0$

~~e)~~ $f(1) = \infty \neq f(0) = 0$

$\frac{1}{0} = \infty$

$\frac{0}{1} = 0$

4. If $\odot d$ denotes the area of a circle with diameter d , then which of the following is equal to $\odot 4 \cdot \odot 6$?

(A) $\odot 10 \rightarrow r = 5 \rightarrow 25\pi$

(B) $\odot 12 \rightarrow r = 6 \rightarrow 36\pi$

(C) $\odot 24 \rightarrow r = 12 \rightarrow 144\pi$

(D) $\pi \cdot \odot 12 \rightarrow r = 6 \rightarrow 36\pi \cdot \pi$

(E) $\pi \cdot \odot 24 \rightarrow r = 12 \rightarrow 144\pi \cdot \pi$

$\odot 4 \rightarrow r = 2 \rightarrow 4\pi$

$\odot 6 \rightarrow r = 3 \rightarrow 9\pi$

$\odot 4 \cdot \odot 6 = 36\pi \cdot \pi$

If $\boxed{x} = (x+2)x$, for all x , what is the value of $\boxed{x+2} - \boxed{x-2}$?

- (A) -2
(B) $x+4$
(C) 0
(D) x^2
(E) $8(x+1)$

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Questions 15–16 refer to the following definition:

Define the symbol $*$ by the following equation: $x^* = 2 - x$, for all non-negative x .

15. $(a + b^*)^* =$

- (A) $b - a$
(B) $a - b - 4$
(C) $b - a + 4$
(D) $a + b - 2$
(E) $a - b$

16. If $(2 - x)^* = (x - 2)^*$, then $x =$

- (A) 0

Questions 15–16 refer to the following definition:

Define the symbol $*$ by the following equation: $x^* = 2 - x$, for all non-negative x .

15. $(a + b^*)^* =$

- (A) $b - a$
- (B) $a - b - 4$
- (C) $b - a + 4$
- (D) $a + b - 2$
- (E) $a - b$

16. If $(2 - x)^* = (x - 2)^*$, then $x =$

- (A) 0
- (B) 1
- (C) 2
- (D) 4
- (E) 6

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Mixture Problems

$$C_I \times (V_I) + C_A \times (V_A) = C_D \times (V_T)$$

C_I Concentration initial

V_I Volume initial

C_A Concentration added

V_A Volume added

C_D Concentration desired

V_T Volume total

If a farmer has 3 pounds of fertilizer that is 5% pesticide, how many pounds of fertilizer with 20% pesticide must he add to create a new fertilizer with 15% pesticide?

$$\begin{aligned} \overbrace{5\%}^{\text{Pest}} (3) + \overbrace{20\%}^{\text{Pest}} (x) &= \overbrace{15\%}^{\text{Pest}} (3+x) \quad \cdot 100 \\ 5x &= 10(3) \\ x &= 6 \end{aligned}$$

A florist wishes to make bouquets of mixed spring flowers. Each bouquet is to be made up of tulips at \$30 a bunch and daisies at \$21 a bunch. How many bunches of each should she use to make 15 bunches which she can sell for \$24 a bunch?

$$\begin{array}{c} 15 \\ \swarrow \quad \searrow \\ x \quad 15-x \end{array}$$

$$\begin{aligned} 30(x) + 21(15-x) &= 24(15) \\ 9x &= 3(15) \\ x &= 5 \end{aligned}$$

A service station checks Andy's radiator and finds it contains only 30% antifreeze. If the radiator holds 10 quarts and is full, how much must be drained off and replaced with pure antifreeze in order to bring it up to required 50% antifreeze?

$$\begin{aligned} 30(10) - 30(x) + 100(x) &= 50(10) \\ 70x &= 20(10) \\ x &= \frac{20}{7} \end{aligned}$$

A florist wishes to make bouquets of mixed spring flowers. Each bouquet is to be made up of tulips at \$30 a bunch and daisies at \$21 a bunch. How many bunches of each should she use to make 15 bunches which she can sell for \$24 a bunch?

$$30(x) + 21(y) = 24(x+y)$$

$$30(\dot{x}) + 21(\dot{y}) = 24(\dot{x} + \dot{y})$$

$$6x = 3y$$

$$2x = y$$

$$x : y$$

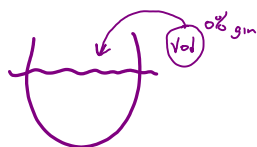
$$1 : 2$$

$$5 : 10$$

Total

$$3$$

$$15$$



35. A drink contains 40% gin and 60% vodka. If the volume of the drink is (n) how much vodka must be added in terms of (n) to be left with a 25% concentration of gin?

$$40\% \left(n \right) + 0\% \left(x \right) = 25\% \left(n + x \right)$$

$$15n = 25x$$

$$\frac{15}{25} n = x$$

$$\frac{3}{5} n = x$$

$$60 \left(n \right) + 100 \left(x \right) = 75 \left(n + x \right)$$

$$25x = 15n$$

$$x = \frac{3}{5} n$$

36. Solution Y is 30 percent liquid X and 70 percent water. If 2 kilograms of water evaporate from 8 kilograms of solution Y and 2 kilograms of solution Y are added to the remaining 6 kilograms of liquid, what percent of this new solution is liquid X?

$$30\% \left(8 \right) - 0\% \left(2 \right) + 30\% \left(2 \right) = x\% \left(8 \right)$$

$$30(10) = 8x$$

$$37.5 = x$$

37. A room contains 160 people, 15% of whom are women. A group of people, 30% of whom are women, leaves the room. Of the people remaining in the room, 10% are women. How many people left the room?

$$15\% \left(160 \right) - 30\% \left(x \right) = 10\% \left(160 - x \right)$$

$$5(160) = 20x$$

$$40 = x$$

38. Seed mixture X is 40 percent ryegrass and 60 percent bluegrass by weight; seed mixture Y is 25 percent ryegrass and 75 percent fescue. If a mixture of X and Y contains 30 percent ryegrass, what percent of the weight of this mixture is X?

17.

17.

17.

Y is 25 percent ryegrass and 75 percent fescue. If a mixture of X and Y contains 30 percent ryegrass, what percent of the weight of this mixture is X?

$$\overset{R_y}{40\%}(X) + \overset{R_y}{25\%}(Y) = \overset{R_y}{30\%}(X+Y)$$

$$\overset{R}{40\%}(X) + \overset{R}{25\%}(100-X) = \overset{R}{30\%}(100)$$

$$15X = 5(100)$$

$$3X = 100$$

$$X = 33.3$$

$$10X = 5Y$$

$$2X = Y$$

$$X = Y \quad \text{Total}$$

$$1:2 \quad 3$$

$$\frac{1}{3} = 33\%$$

39. There are two alloys of copper and zinc. In the first alloy, the ratio of copper to zinc is 3:4 and in the second alloy the ratio of copper to zinc is 6:1. In what proportion should these two alloys be mixed so that a new alloy containing equal parts of copper and zinc may be obtained?

$$\frac{3}{7} (x) + \frac{6}{7} (y) = \frac{1}{2} (x + y) \quad \left. \vphantom{\frac{3}{7} (x) + \frac{6}{7} (y) = \frac{1}{2} (x + y)} \right] \cdot 14$$

$$6x + 12y = 7x + 7y$$

$$5y = x$$

$$x = y$$

$$5 = 1$$

Number Properties

40. What is the units digit of $3^{17} \times 17^{54} \times 135^{19} \times 31^{21} \times 42^{17}$?

\downarrow \downarrow \downarrow \downarrow \downarrow
 odd odd 5 odd even

ends with zero

How many zeros does it end with?

17 zeros

$135^{19} = (3^3 \cdot 5)^{19} = 3^{57} \cdot 5^{19}$
 $42^{17} = (2 \cdot 3 \cdot 7)^{17} = 2^{17} \cdot 3^{17} \cdot 7^{17}$
 $3^{57} \cdot 5^{19} \cdot 2^{17} \cdot 3^{17} \cdot 7^{17}$
 $= 2^{17} \cdot 3^{74} \cdot 5^{19} \cdot 7^{17}$
 $= 2^{17} \cdot 5^{17} \cdot 5^2 \cdot 10^{17}$

42^{17}

$2^1 \rightarrow 2$
$2^2 \rightarrow 4$
$2^3 \rightarrow 8$
$2^4 \rightarrow 6$
$2^5 \rightarrow 2$
$2^{17} \rightarrow$

 $\frac{17}{4} R=1$
 [2]

41. Which of the following fractions has a decimal equivalent that is a terminating decimal?

a) 10/189

b) 15/196

c) 16/225

~~d) 25/144~~

e) 39/256

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$$\begin{array}{r} 39 \\ \hline 256 \end{array}$$

$$\frac{25}{2^4 \cdot 3^2}$$

yuck

In such problems look only
in the denominator

Now terminate

$$\frac{1}{3} = 0.333\bar{3}$$

Terminate

$$\frac{1}{4} = 0.25$$

all numbers with
a 2^n and/or 5^n only
in a denominator

Terminate

$$\frac{3}{2^7 \cdot 5^3}$$

42. If $n = 8^{111} - 8$, what is the units digit of n ?

8^1	\rightarrow	8
8^2	\rightarrow	4
8^3	\rightarrow	2
8^4	\rightarrow	6
8^5	\rightarrow	8

$$\frac{111}{4} \rightarrow R = 3$$

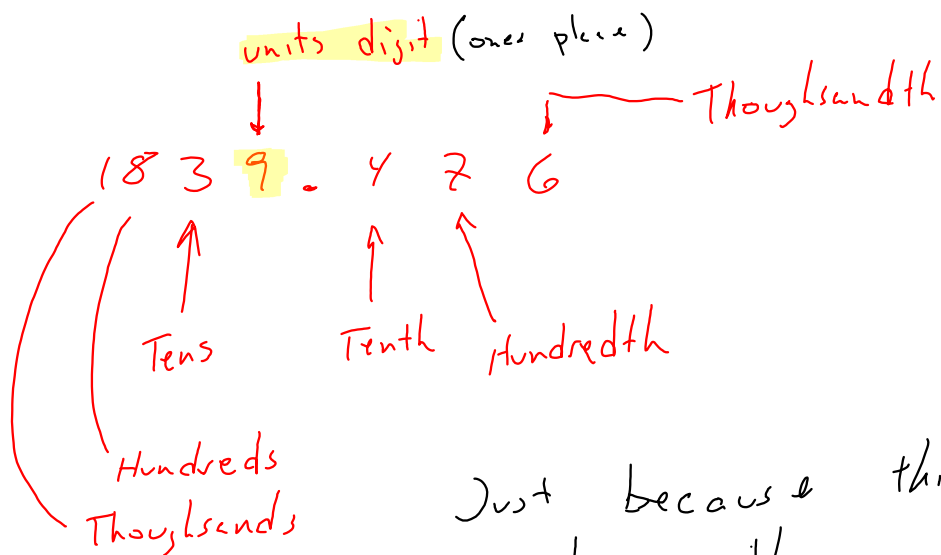
Take any number of any size that ends with a 2 !!! Then subtract 8. Ah it ends with a 4

$$1342 - 8 = 1334$$

$$93852 - 8 = 93844$$

$$8^{111} - 8$$

8^{111} is a really really really big number that ends with a two.



Just because this number
ends with a 6 it
is not the units digit.
The units digit = ones place.

43. If M and N are positive integers that have remainders of 1 and 3, respectively, when divided by 6, which of the following could NOT be a possible value of $M+N$?

a) 86

b) 52

c) 34

d) 28

e) 10

$$\frac{M}{6}$$

$$R = 1$$

$$M \equiv 1$$

7

$$\frac{13}{6}$$

19

$$\frac{25}{6}$$

$$\frac{13}{4}$$

Q

R

3

1

0

1

$$\frac{N}{6}$$

$$R = 3$$

$$N \equiv 3$$

9

$$\frac{15}{6}$$

21

$$\frac{27}{6}$$

43. If M and N are positive integers that have remainders of 1 and 3, respectively, when divided by 6, which of the following could NOT be a possible value of $M+N$?

a) 86

b) 52

c) 34

d) 28

e) 10

$$\frac{M+N}{6}$$

$$R = 4$$

$$\frac{10}{6}$$

$$R = 4$$

$$\frac{28}{6}$$

$$R = 4$$

$$\frac{34}{6}$$

$$R = 4$$

$$\frac{52}{6}$$

$$R = 4$$

$$\frac{86}{6}$$

$$R = 2$$

44. When the integer n is divided by 17, the quotient is x and the remainder is 5. When n is divided by 23, the quotient is y and the remainder is 14. Which of the following is true?

- a) $23x + 17y = 19$
- ☒ b) $17x - 23y = 9$
- c) $17x + 23y = 19$
- d) $14x + 5y = 6$
- e) $5x - 14y = -6$

$$\frac{N}{D} \quad Q \quad R$$

$$\frac{13}{4} \quad 3 \quad 1$$

$$13 = 4 \cdot 3 + 1$$

$$N = D \cdot Q + R$$

$$\frac{n}{17} \quad x \quad 5$$

$$n = 17x + 5$$

$$\frac{n}{23} \quad y \quad 14$$

$$n = 23y + 14$$

$$17x + 5 = 23y + 14$$

$$17x - 23y = 9$$

45. When N is divided by 10 the remainder is 1 and when N is divided by 3 the remainder is 2. What is the remainder when N is divided by 30?

$$\frac{N}{10} \quad R = 1 \quad N = ? \quad 1 \quad 11 \quad 21 \quad 31 \dots$$

$$\frac{N}{3} \quad R = 2 \quad N = ? \quad 2 \quad 5 \quad 8 \quad 11$$

$$N = 11$$

$$\frac{N}{30}$$

$$R = 11$$

46. What is the remainder when 2^{91} is divided by 7?

$$\begin{array}{r} 2^1 \\ \hline 7 \end{array}$$
$$R = 2$$
$$\begin{array}{r} 2^2 \\ \hline 7 \end{array}$$
$$R = 4$$
$$\begin{array}{r} 2^3 \\ \hline 7 \end{array}$$
$$R = 1$$

$$\begin{array}{r} 2^4 \\ \hline 7 \end{array}$$

$$R = 2$$

$$\frac{91}{3} \quad R = 1$$

47. If $y+2$ is divisible by 4 then is $y/2$ odd or even?

$y + 2 =$	4	8	12
$y =$	2	6	10
$\frac{y}{2} =$	1	3	5
	odd	odd	odd

Dont insert for y
Insert for $y+2$
= mult of 4
(That's the fact)

← always odd

48. If k is an even integer and p and r are odd integers, which of the following cannot be an integer?

- a) r/k
- b) k/p
- c) p/r
- d) kp/r
- e) kr/p

$\frac{\text{even}}{\text{odd}}$

might be an integer $\left(\frac{2 \cdot 3}{3}\right)$

$\frac{\text{odd}}{\text{even}}$

never an integer $\left(\frac{3 \cdot 7 \cdot 11}{2 \cdot 3}\right)$

(a) $\frac{\text{odd}}{\text{even}}$

49. How many divisors does 39690 have?

$$\begin{array}{c}
 3969 \cdot 10 \\
 \wedge \\
 441 \cdot 9 \\
 \wedge \\
 9 \cdot 49 \\
 \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 \textcircled{4} \quad \textcircled{2} \quad \textcircled{1} \quad \textcircled{1} \\
 3 \cdot 7 \cdot 2 \cdot 5 \\
 \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 5 \cdot 3 \cdot 2 \cdot 2 = 60
 \end{array}$$

$$\begin{array}{r}
 441 \\
 9 \overline{) 3969} \\
 \underline{36} \\
 36
 \end{array}$$

$$\begin{array}{c}
 72 \\
 \wedge \\
 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \\
 \textcircled{2} \quad \textcircled{3} \\
 3 \cdot 2 \\
 \downarrow \quad \downarrow \\
 3 \cdot 4 = 12
 \end{array}$$

1	72
2	36
3	24
4	18
6	12
8	9

12 distinct factors

50. Which of the following is not a prime number?

- a) 323
- b) 263
- c) 241
- d) 197
- e) 131

263 → ~~13~~
~~11~~
 Prime ~~7~~
~~5~~
~~3~~
~~2~~

241 → ~~13~~
~~11~~
 Prime ~~7~~
~~5~~
~~3~~
~~2~~

197 ~~13~~
~~11~~
 Prime ~~7~~
~~5~~
~~3~~
~~2~~

131 ~~11~~
~~7~~
 Prime ~~5~~
~~3~~
~~2~~

To determine if a number is prime
 test the divisibility of all primes less
 than the root of the number.

If none of these primes are factors
 the number is prime.

51. What is the least common multiple of 8, 9, 10, 11, 12 and 24?

LCM

2 2 2

3 3

2 5

11

~~2~~ ~~2~~ ~~3~~

~~2~~ ~~2~~ ~~2~~ ~~3~~

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 11$

3960

HCF

18 and 12
 $(2 \cdot 3) \cdot 3$ $2 \cdot (2 \cdot 3)$
→ 6

52. When the product of 3070956 and n divided by 720 there will be no remainder. If $n > 0$, what is the smallest value of n ?

Handwritten solution:

$2 \cdot 2 \cdot 3$

3070956 (x)

~~2~~ · ~~2~~ · 2 · 2 · ~~3~~ · 3 · 5

60

53. How many of the positive factors of 42 are not factors of 56?

a) 1

b) 2

c) 3

d) 4

e) 5

42

$2 \cdot 3 \cdot 7$

1

2

3

6

42

21

14

7

56

$2 \cdot 2 \cdot 2 \cdot 7$

1

2

4

7

56

28

14

8

54. What is the smallest positive integer n for which 324 is a factor of 6^n ?

- ~~a) 2~~
- ~~b) 3~~
- c) 4
- d) 5
- e) 6

$$\frac{6^n}{324}$$

$$\frac{2^n \cdot 3^n}{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$$

mult
—
fac

$$\frac{\cancel{2} \cancel{2} \cancel{2} \cancel{2} \cancel{3} \cancel{3} \cancel{3} \cancel{3}}{\cancel{2} \cancel{2} \cancel{3} \cancel{3} \cancel{3} \cancel{3}}$$

55. $10^{25} - 560$ is divisible by all the following EXCEPT?

- a) 11
- ~~b) 8~~
- ~~c) 5~~
- ~~d) 4~~
- e) 3

$$\begin{array}{r}
 1000 \dots 0000 \\
 - \quad \quad \quad 560 \\
 \hline
 \dots 99 \quad 440 \\
 \hline
 22
 \end{array}$$

is it a multiple of 11

$$\begin{array}{r}
 A \ B \ A \\
 7 \ 5 \ 9
 \end{array}$$

or

$$\Sigma A = 16$$

$$\Sigma B = 5$$

$$11$$

AB OK

$$99$$

$$A - B : 0$$

$$\begin{array}{r}
 A \ B \ A \ B \ A \\
 3 \ 7 \ 9 \ 5 \ 6
 \end{array}$$

$$A = 18$$

$$B = 12$$

$$6$$

Not a multiple

Divisibility Rules

A number is divisible by 2 if it is even.

A number is divisible by 4 if its last 2 digits are a multiple of 4.

A number is divisible by 8 if its last 3 digits are divisible by 8 (look for blocks of 200).

A number is divisible by 3 if the sum of all its digits is a multiple of 3.

A number is divisible by 9 if the sum of all its digits is a multiple of 9.

A number is divisible by 5 if it ends with a 5 or zero.

A number is divisible by 10 if it ends with a zero.

A number is divisible by 11 by rule of ABABA.

If $\Sigma A - \Sigma B$ is zero or a multiple of 11 then the number is a multiple of 11.

In 374 $\Sigma A = 7$ and $\Sigma B = 7$ and $\Sigma A - \Sigma B = 0$, therefore 374 is a multiple of 11.

A	B	A
3	7	4

A number is divisible by 6 if it has at least one factor of 2 and at least one factor of 3.

A number is divisible by 12 if it has at least two factors of 2 and at least one factor of 3.

A number is divisible by 18 if it has at least one factor of 2 and at least two factors of 3.

A number is divisible by 24 if it has at least three factors of 2 and at least one factor of 3.

A number is divisible by 36 if it has at least two factors of 2 and at least two factors of 3.

BLOCK METHOD

For any number which you do not know a rule think BLOCKS.

Is 263 divisible by 13? Well 130 is. 260 is. And the difference between 260 and 263 is 3, which is not a multiple of 13 so 263 is not divisible by 13.

Note:

if a number is divisible by 2 it has at least 1 factor of 2

if a number is divisible by 4 it has at least 2 factors of 2

if a number is divisible by 8 it has at least 3 factors of 2

if a number is divisible by 4 and not by 8 it has exactly 2 factors of 2

if a number is divisible by 3 it has at least 1 factor of 3

if a number is divisible by 9 it has at least 2 factors of 3

if a number is divisible by 3 and not by 9 it has exactly 1 factors of 3

56. What is the smallest integer that 10584 must be multiplied by so that the resulting number is a perfect cube?

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$$\begin{array}{r} 147 \\ 8 \overline{) 1176} \\ \underline{8} \\ 37 \\ \underline{32} \\ 56 \end{array}$$

$$\begin{array}{r} 49 \\ 3 \overline{) 147} \\ \underline{12} \\ 27 \end{array}$$

$$\begin{array}{r} 1176 \\ 9 \overline{) 10584} \\ \underline{9} \\ 158 \\ \underline{15} \\ 138 \\ \underline{135} \\ 34 \end{array}$$

$$3 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \cdot 7 \cdot 7 \cdot 7$$

If you were asked same question but so the result is a perfect square?

$$3 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \cdot 7 \cdot 7 \cdot 3 \cdot 2$$

57. By using the numbers 1,2,3,5 and 7 only once, how many five digit numbers can be made that are divisible by 25?

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$$\begin{array}{l} \boxed{137} \underline{25} \quad 3! \\ \boxed{132} \underline{75} \quad + 3! \\ \hline (12) \end{array}$$

SAME Question but numbers 0,2,3,5,7

$$\begin{array}{l} \boxed{073} \underline{25} \quad 4 \\ \boxed{237} \underline{50} \quad 6 \\ \boxed{032} \underline{75} \quad 4 \\ \hline (14) \end{array}$$

Number must be a multiple of both 9 and 8

58. What is the smallest sum of integers m and n so that the five digit number $7m1n0$ is divisible by 72?

$$\begin{array}{lcl}
 n = 2 & \rightarrow & 7m120 \\
 m = 8 & \rightarrow & 78120 \\
 \hline
 & & \textcircled{10}
 \end{array}
 \quad \sum \text{ of dig} = \frac{18}{9}$$

59. How many positive integers less than 1000 have no factors (other than 1) in common with 1000?

- a) 400
- b) 410
- c) 411
- d) 412
- e) None of the above

$$10 \cdot 10 \cdot 10 \\ 2 \cdot 5 \quad 2 \cdot 5 \quad 2 \cdot 5$$

since the prime factors of 1000 are $2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$ any number that also has a prime factor of 2 or 5 would have a factor in common with 1000.

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Thus a number that ends with 5 or an even thus has a factor in common.

Question is really how many numbers less than 1000 do not end with an even or 5?

$$\begin{array}{r} 10 \\ \hline 0-9 \end{array} = \begin{array}{r} 10 \\ \hline 0-9 \end{array} \cdot \begin{array}{r} 4 \\ \hline 1,3,7,9 \end{array} = \textcircled{400}$$

9	9	9
0	0	1

60. The sum of two numbers is 588 and their HCF is 49. How many such pairs of numbers can be formed?

$$X + Y = 588$$

$$49a + 49b = 588 \quad \div 49$$

$$a + b = 12$$

a	b
1	11
2	10
3	9
4	8
5	7
6	6

$$X = 49(a)$$

$$Y = 49(b)$$

X	Y
49 · 1	49 · 11
49 · 2	49 · 10
49 · 6	49 · 6

2

61. How many zeros does 100! end with?

$$100! = 2^{50+} \cdot 5^{24} \cdot Z$$

$100!$
 $5^1 \rightarrow 20$
 $5^2 \rightarrow 4$

24

$2^{24} \cdot 5^{24}$
 10^{24}

24 zeros

62. What is the highest integer power of 12 that divides $27!$ evenly?

$$\frac{27!}{12^n}$$

$$\frac{27!}{2^n \cdot 2^n \cdot 3^n}$$

$$n = 11$$

$$27!$$

$$\begin{array}{r} 2^1 \quad 13 \\ 2^2 \quad 6 \\ 2^3 \quad 3 \\ 2^4 \quad 1 \\ \hline 23 \end{array}$$

$$27!$$

$$\begin{array}{r} 3^1 \quad 9 \\ 3^2 \quad 3 \\ 3^3 \quad 1 \\ \hline 13 \end{array}$$

$$2^{11} \cdot 2^{11} \cdot 3^{11}$$

22 ok

$n = 12$ Too Big

$$12 + 12 = 24$$

$$27! = 2^{23} \cdot 3^{13} \cdot 5 \cdot \dots$$

$$\begin{array}{c} \cancel{2^{12} \cdot 2^{12} \cdot 3^{12}} \\ \hline 2^{11} \cdot 2^{11} \cdot 3^{11} \end{array}$$

$$n = 11$$

63. Which is bigger $89! - 88!$ or $87! \times 88^2$?

$$88! (89 - 1)$$

$$88! \cdot 88$$

$$87! \cdot 88 \cdot 88$$

$$87! \cdot 88^2$$

They are equal.

64. If n is the product of the integers from 1 to 20 inclusive, which of the following is the greatest integer k for which $2k$ is a factor of n ?
- a) 408
 - b) 437
 - c) 486
 - d) 532
 - e) 1242

$$\frac{20!}{2k}$$

$$\frac{20!}{2 \cdot 3 \cdot 3 \cdot 2 \cdot 3 \cdot 2 \cdot 3}$$

$$\begin{array}{r} 133 \\ 4 \overline{) 532} \\ \underline{4} \\ 13 \\ \underline{12} \\ 12 \end{array}$$

$$\begin{array}{r} 137 \\ 9 \overline{) 1242} \\ \underline{9} \\ 34 \\ \underline{27} \\ 72 \\ \underline{72} \\ 0 \end{array}$$

$$\begin{array}{r} 138 \\ 6 \overline{) 138} \\ \underline{12} \\ 18 \end{array}$$

$$3 \cdot 3 \cdot 2 \cdot 3 \cdot 2 \cdot 3$$

$$\begin{array}{r} 19 \\ 7 \overline{) 133} \\ \underline{7} \\ 63 \end{array}$$

$$\frac{20!}{2 \cdot 2 \cdot 2 \cdot 7 \cdot 19}$$

$$\frac{x}{22} \quad ? \quad \text{or} \quad \frac{x}{2 \cdot 11} \quad ?$$

65. Is 22 a factor of x?
(1) 22 is a factor of 15x
(2) 22 is a factor of 16x

- (1) $\frac{3 \cdot 5 \cdot x}{2 \cdot 11}$; yes

A

~~(2)~~ $\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot x}{2 \cdot 11}$; maybe

Percents

66. In addition to the price of the meal, Janet paid sales tax equal to 8.5% of the price of the meal, and a tip equal to 15% of the price of the meal. If she paid \$2.04 in sales tax, how much was the tip?

$$8.5\% (m) = 204$$

$$\text{Tip} = 15\% (m)$$

$$m = \frac{204}{8.5\%}$$

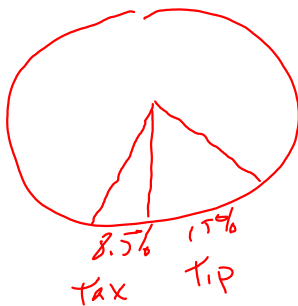
$$\text{Tip} = 15\% \cdot \frac{204}{8.5\%} = \frac{15 \cdot 204 \cdot 2}{8.5 \cdot 2} = \frac{15 \cdot 204 \cdot 2}{17}$$

$$= 15 \cdot 12 \cdot 2$$

$$= 360$$

$$= \$3.60$$

method Ratio



$$\frac{8.5}{204} = \frac{15}{x}$$

$$8.5x = 204 \cdot 15$$

$$x = \frac{204 \cdot 15}{8.5}$$

$$= \frac{204 \cdot 15 \cdot 2}{17}$$

$$= 12 \cdot 15 \cdot 2$$

$$\boxed{= 360}$$

67. During a four-day sale, a store sells 20% of its stock of kitchen tables on the first day, 25% of the remaining stock on the second day and one-third of the unsold tables on the third day and 50% of what was left on the last day. If 18 tables remained after the sale was over, how many tables were in the stock at the beginning of the sale?

$$x \cdot \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} = 18$$

$$\frac{x}{5} = 18$$

$$x = 90$$

$$100 \xrightarrow[-20\%]{20} 80 \xrightarrow[-25\%]{20} 60 \xrightarrow[-33\%]{20} 40 \xrightarrow[-50\%]{20} 20$$

68. Two items are sold for 1200 each if the merchant made a profit of 25% on one and a loss of 25% on the other, what is his overall profit or loss?

Profit

$$125\% \text{ CP} = 1200 \rightarrow \left. \begin{array}{l} \frac{5}{4} \text{ CP} = 1200 \\ \text{CP} = \frac{1200 \cdot 4}{5} \\ \text{CP} = 960 \end{array} \right\} \begin{array}{l} \text{Profit} \\ +240 \end{array}$$

Loss

$$75\% \text{ CP} = 1200 \rightarrow \left. \begin{array}{l} \frac{3}{4} \text{ CP} = 1200 \\ \text{CP} = \frac{1200 \cdot 4}{3} \\ \text{CP} = 1600 \end{array} \right\} \begin{array}{l} \text{Loss} \\ 400 \end{array}$$

-160

$$\text{CP} = \text{SP} \rightarrow \text{Profit/Loss} = 0$$

$$110\% \text{ CP} = \text{SP} \rightarrow \text{Profit } 10\%$$

$$90\% \text{ CP} = \text{SP} \rightarrow \text{Loss } 10\%$$

69. Mr. Johnson invested \$50,000 into two investments. Part of it he put in a gold mine stock from which he hoped to receive a 20% return per year. The rest he invested in a bank stock which was paying 6 percent per year. If he received \$400 more the first year from the bank stock than from the mining stock, how much did he invest in each stock?

$$\begin{aligned}
 B + G &= 50000 \\
 400 + \underbrace{20\% G}_{\text{earnings gold}} &= \underbrace{6\% B}_{\text{earnings Bank}} \quad] \cdot 100 \\
 40000 + 20G &= 6B \\
 -6B + 20G &= -40000 \\
 6B + 6G &= 300000 \quad \leftarrow \\
 76G &= 260000 \\
 G &= 10000 \\
 B &= 40000
 \end{aligned}$$

69. Mr. Johnson invested \$50,000 into two investments. Part of it he put in a gold mine stock from which he hoped to receive a 20% return per year. The rest he invested in a bank stock which was paying 6 percent per year. If he received \$400 more the first year from the bank stock than from the mining stock, how much did he invest in each stock?

earn from gold

Screen capture taken: 20.12.2012 09:22

$$\begin{aligned}
 20\%(x) + 400 &= 6\%(50000 - x) \\
 20\%(x) + 400 &= 3000 \\
 20\%(x) &= 2600 \\
 x &= 10000 \\
 50000 - x &= 40000
 \end{aligned}$$

earn from bank

BOB is 4 years older than Nancy

$$B = N + 4$$

Am't earned from bank is 400 more than am't. earned from Gold.

$$B = G + 400$$

70. If an item is sold at 80% of its present selling price, the merchant will make a loss of 4%. What percent profit does he make by selling at the current selling price?

CP	SP	DP
100		96

+20%

$$80\% \text{ SP} = 96$$

$$\text{SP} = \frac{96 \cdot 100}{80} = 120$$

10. A merchant sells an item at a 20% discount, but still makes a profit of 20 percent of the cost. What percent of the cost would the profit on the item have been if it had been sold without the discount?

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CP	SP	DP
100		120

+20%

$$80\% \text{ SP} = 120$$

$$\text{SP} = \frac{120 \cdot 100}{80} = 150$$

+50%

71. A merchant marks his goods in such a way that his profit on sale of 50 items equals the selling price of 10 items. What percent profit does he make?

$$\begin{array}{ccc}
 50 (P) & = & 10 (SP) \\
 \downarrow & & \downarrow \\
 1 & & 5
 \end{array}
 \qquad CP = 4$$

$$\begin{array}{ccc}
 & +1 & \\
 4 & \xrightarrow{\quad} & 5
 \end{array}$$

$$\frac{1}{4} = 25\%$$

The cost price of 20 articles is equal to the selling price of 15 articles. Find the percent profit or loss?

$$\begin{array}{ccc}
 20 CP & = & 15 SP \\
 \downarrow & & \downarrow \\
 3 & & 4
 \end{array}$$

$$\begin{array}{ccc}
 & +1 & \\
 3 & \xrightarrow{\quad} & 4
 \end{array}$$

$$\frac{1}{3} \rightarrow 33.33\%$$

72. Bob bought a goat. He expected to sell it at a price that would give him a 10% profit on his purchase. However, he had to sell it for \$50 less than he expected, which was a loss of 15% on what it cost him. What did Bob pay for the goat?

$$110\% \text{ CP} = \text{SP}$$

$$85\% \text{ CP} = \text{SP} - 50$$

$$25\% \text{ CP} = 50$$

$$\text{CP} = 200$$

$$\text{SP} = 220$$

A used car dealer sold a car at a loss of 10%. If he had sold it for \$5000 more, he would have made a profit of 10%. How much did he sell it for?

$$90\% \text{ CP} = \text{SP}$$

$$110\% \text{ CP} = \text{SP} + 5000$$

$$20\% \text{ CP} = 5000$$

$$\text{CP} = 25000$$

$$\boxed{\text{SP} = 22500}$$

$$90\% \cdot 25000 = \text{SP}$$

$$22500 \quad \text{SP}$$

$$10\% \cdot 25000$$

$$2500$$

$$25000 - 2500$$

$$22500$$

73. The interest compounded annually on a certain sum of money is \$41.60 after two years.
The simple interest on the same amount of money after two years would be \$40.00.
Find the amount invested if the interest rate would be the same in each investment.

20	20	40	} 1.60
	1.60	41.60	

$$x\% (20) = 1.60$$

$$x\% = \frac{1.60}{20}$$

$$x = \frac{160}{24} = 8$$

$$8\% (x) = 20$$

$$x = \frac{20 \cdot 100}{8} = 250$$

How much interest will \$2000 earn at an annual rate of 8% in one year if the interest is compounded every 6 months?

- a) \$160 b) \$163.20 c) \$249.73 d) \$332.80 ~~e) \$2163.20~~ TRAP

Simple
Interest

$$2000 \cdot 1.04^2$$

$$2000 \cdot 1.0816$$

$$2163.20$$

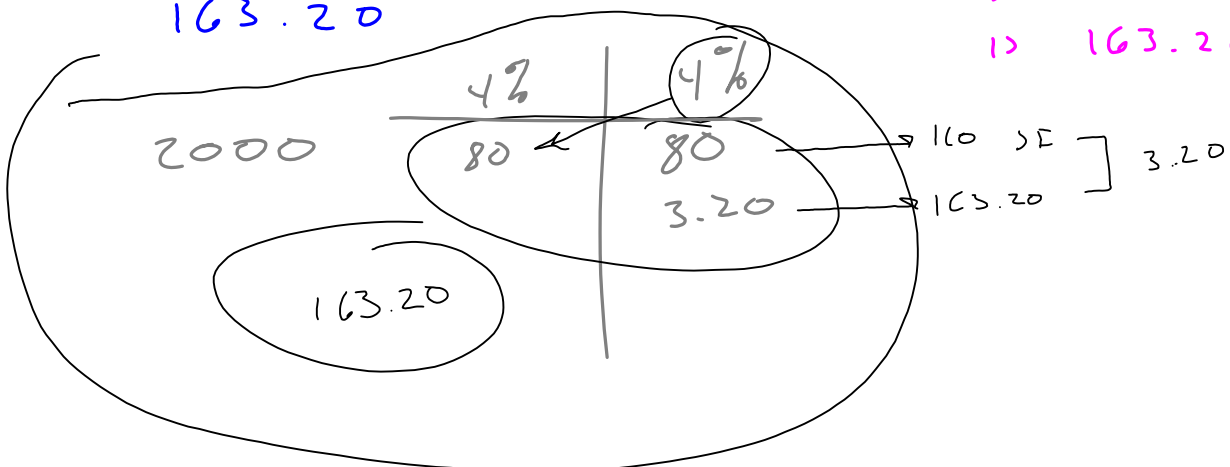
$$\begin{array}{r} 2163.20 \\ - 2000 \\ \hline 163.20 \end{array}$$

8% twice
TRAP

NOTE

- Compound interest is always just a little bit more than simple interest

8% of 2000 = 160
so correct answer
is 163.20 (B)



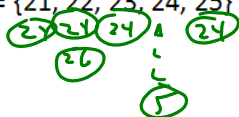
1000 1% for year compounded
every 4 months
 $1000 \cdot 1.02^3$

Part
Whole

74. If any number from set A is multiplied by any number from set B, what is the probability that the product is a multiple of 4?

A = {21, 22, 23, 24, 25}

B = {23, 24, 25, 26, 27}



$$\frac{10}{25} = \frac{2}{5}$$

even · odd has at least
one factor of 2
mult of 4 · any has at least
2 factors of 2
even · even has at least
2 factors of 2

75. A bowl contains 10 apples, 2 of which are bad. If someone randomly selected four apples from the bowl, what is the probability that at least one of the apples is bad?

$$1 - \frac{8}{10} \cdot \frac{7}{9} \cdot \frac{6}{8} \cdot \frac{5}{7}$$

$$1 - \frac{1}{3} = \frac{2}{3}$$

$$\begin{array}{c} G \quad G \quad C \quad B \quad \text{or} \quad G \quad G \quad B \quad B \\ \frac{8}{10} \cdot \frac{7}{9} \cdot \frac{6}{8} \cdot \frac{2}{7} \cdot \frac{4!}{3!} + \frac{8}{10} \cdot \frac{7}{9} \cdot \frac{2}{8} \cdot \frac{1}{7} \cdot \frac{4!}{2!2!} \end{array}$$

Conditional Probability

- 1) pick an order
- 2) solve for that order
- 3) multiple by permutations

A jar contains 6 red balls and 4 blue balls. If you reached in and took four balls out in one grab (no replacement), what is the probability that two balls would be blue and the other two would be red?

$$\begin{array}{c} B \quad B \quad R \quad R \\ \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{6}{8} \cdot \frac{5}{7} \cdot \frac{4!}{2!2!} \end{array}$$

In a five day period the chance of rain each day is 25% what is the chance that it will rain only twice?

$$\begin{array}{c} R \quad R \quad N \quad N \quad N \\ \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{5!}{2!3!} \end{array}$$

75. A bowl contains 10 apples, 2 of which are bad. If someone randomly selected four apples from the bowl, what is the probability that at least one of the apples is bad?

$$1 - G G G G \quad \leftarrow \text{Best approach}$$

$$1 - \frac{8}{10} \cdot \frac{7}{9} \cdot \frac{6}{8} \cdot \frac{5}{7}$$

$$1 - \frac{1}{3} = \frac{2}{3}$$

Direct approach

G G B B

or

G G G B

$$\frac{8}{10} \cdot \frac{7}{9} \cdot \frac{2}{8} \cdot \frac{1}{7} \cdot \frac{4!}{2!2!}$$

+

$$\frac{8}{10} \cdot \frac{7}{9} \cdot \frac{6}{8} \cdot \frac{2}{7} \cdot \frac{4!}{3!}$$

$$= \frac{2}{3}$$

76. In a five day period the chance of rain each day is 50% what is the chance that it will rain only twice?

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{5!}{3!2!}$$

77. The probability of picking two red balls from a container of red and blue balls is $\frac{2}{9}$. If there are 5 blue balls in the container, how many red balls were in the container?

a) 4

b) 5

c) 6

d) 7

e) 8

method B.S.

Red
~~7~~

$$\frac{7}{12} \cdot \frac{6}{11} \neq \frac{2}{9}$$

→ 5

$$\frac{5}{10} \cdot \frac{4}{9} = \frac{2}{9}$$

method too much time

$$\frac{R}{R+5} \cdot \frac{R-1}{R-1+5} = \frac{2}{9}$$

$$\frac{R}{R+5} \cdot \frac{R-1}{R+4} = \frac{2}{9}$$

$$\frac{R^2 - R}{R^2 + 5R + 4R + 20} = \frac{2}{9}$$

$$9(R^2 - R) = 2(R^2 + 9R + 20)$$

$$9R^2 - 9R = 2R^2 + 18R + 40$$

$$7R^2 - 27R - 40 = 0 \quad \leftarrow \text{yuck!!}$$

78. The probability of a man hitting a bull's-eye in one throw at a dart board is $\frac{1}{4}$. What is the least number of throws he must take in order for the probability of hitting the bulls eye at least once to be more than 50%?

$$1 - \frac{1^N}{4} = \frac{1}{4}$$

$$1 - \frac{1^N}{4} \cdot \frac{1^N}{4} = 1 - \frac{1}{16} = \frac{15}{16}$$

$$1 - \frac{1^N}{4} \cdot \frac{1^N}{4} \cdot \frac{1^N}{4} = 1 - \frac{1}{64} = \frac{63}{64} > 50\%$$

3

$1 - 1^N 1^N 1^N = \text{getting at least once}$

Ratios

$$\frac{3}{7} \cdot 7 \cdot 13 = 39$$

79. Exactly $\frac{3}{7}$ of the people in the room are under the age of 21, and exactly $\frac{5}{13}$ of the people in the room are over the age of 65. If the total number of people in the room is greater than 50 and less than 100, how many people in the room are under the age of 21?

a) 21
 b) 35
 c) 39
 d) 60
 e) 65

People under 21 can be any multiple of 3
 People over 65 can be any multiple of 5

U21 : REST : 50 < TOTAL < 100
 3 : 4 : 7
 x7 : x13 : x20

21 → 49 (too small)
 39 → 91 (OK)
 60 → 140 (too big)

DUE TO THE RATIO
3:4

Total people under 21 must be a multiple of 3

Total people that represent the rest of the people in the room must be a multiple of 4

The total people in the room must be a multiple of 7.

$$\frac{3}{7} \quad \frac{5}{13}$$

Total people must be a multiple of 7 and 13

First multiple of 7 and 13 is 91
since 91 is between 50 and 100,

we would know there are 91 people
in the room.

$$\frac{3}{7} \cdot 91 = 39$$

$$3x:2x$$

80. A certain bag contains a mixture of nuts and raisins, in the ratio of 3:2, nuts to raisins by weight. If 15 pounds of nuts are removed, and are replaced with 20 pounds of raisins, so that the new ratio is 3:4, how many pounds of raisins were in the original mixture?

$$2x = ?$$

$$\begin{array}{l} n \quad \frac{3x - 15}{2x + 20} = \frac{3}{4} \\ r \end{array}$$

$$12x - 60 = 6x + 60$$

$$6x = 120$$

$$x = 20$$

$$\boxed{2x = 40}$$

The ratio of flowering to nonflowering plants on an acre of land is 3 to 2. After 140 nonflowering plants are removed from the property, the new ratio of flowering to nonflowering plants is 4 to 1. How many plants were there on the acre of land before the 140 nonflowering plants were removed?

$$\begin{array}{l} F + N \\ 3x + 2x = 5x = ? \end{array}$$

$$\begin{array}{l} F \quad \frac{3x}{2x - 140} = \frac{4}{1} \\ N \end{array}$$

$$3x = 8x - 560$$

$$\boxed{560 = 5x}$$

In a certain high school, the ratio of teachers to students is 5 to 108. If 12 new students entered the school, ratio would change to 5 to 112. How many teachers does the high school have?

$$5x = ?$$

$$\begin{array}{l} T \quad \frac{5x}{108x + 12} = \frac{5}{112} \\ S \end{array}$$

$$\cancel{5}x(112) = \cancel{5}(108x + 12)$$

$$112x = 108x + 12$$

$$4x = 12$$

$$x = 3$$

$$\boxed{5x = 15}$$

A car dealer had a sale in order to sell off excess stock. Before the sale, the dealer had three times as many cars as he had trucks. During the sale, he sold 100 cars and 25 trucks. If after the sale the dealer has twice as many cars as trucks, then how many cars did the dealer have prior to the sale?

A car dealer had a sale in order to sell off excess stock. Before the sale, the dealer had three times as many cars as he had trucks. During the sale, he sold 100 cars and 25 trucks. If after the sale the dealer has twice as many cars as trucks, then how many cars did the dealer have prior to the sale?

- a) 60
- b) 90
- c) 120
- ☒ d) 150
- e) 180

$$\begin{array}{l}
 C \\
 T
 \end{array}
 \quad
 \frac{3x - 100}{x - 25} = \frac{2}{1}$$

$$3x - 100 = 2x - 50$$

$$x = 50$$

$$\boxed{3x = 150}$$

Philip has twice as many salamanders as Matt. If Philip gives Matt 10 of his salamanders, he will have half as many as Matt. How many salamanders do Philip and Matt have together?

- a) 10
- b) 20
- c) 30
- d) 40
- e) 60

$$\begin{array}{l}
 P \\
 M
 \end{array}
 \quad
 \frac{2x - 10}{x + 10} = \frac{1}{2}$$

$$3x = ?$$

$$4x - 20 = x + 10$$

$$\boxed{3x = 30}$$

81. Andy, Ben, Carl and Dave bought a car for \$6000. Andy paid half the sum of the other boys, Ben paid one third the sum of the other boys and Carl paid one fourth the sum of the other boys. How much did Dave have to pay?

$$\begin{array}{lll} A : \text{Rest} & \text{Total} & \\ 1 : 2 & 3 & \frac{1}{3} \cdot 6000 = 2000 \leftarrow \text{Andy} \end{array}$$

$$\begin{array}{lll} B : \text{Rest} & \text{Total} & \\ 1 : 3 & 4 & \frac{1}{4} \cdot 6000 = 1500 \leftarrow \text{Ben} \end{array}$$

$$\begin{array}{lll} C : \text{Rest} & \text{Total} & \\ 1 : 4 & 5 & \frac{1}{5} \cdot 6000 = 1200 \leftarrow \text{Carl} \end{array}$$

$$4700$$

$$6000 - 4700 = 1300 \leftarrow \text{Dave}$$

82. Bag A contains red, white and blue marbles such that the red to white marble ratio is 1:3 and the white to blue marble ratio is 2:3. Bag B contains red and white marbles in the ratio of 1:4. Together, the two bags contain 30 white marbles. How many red marbles could be in bag A?

- a) 1 **MUST BE A**
b) 3 **MULTIPLE**
c) 4 **OF 2**
d) 6
e) 8

$$\begin{array}{ccc} 2x & 6x & 9x \\ 6 & 18 & 27 \end{array}$$

$$\begin{array}{ccc} R & : & W & : & B \\ 1 & : & 3 & & \\ & & 2 & : & 3 \\ 2 & : & 6 & : & 9 \\ 4 & : & 12 & : & 18 \\ 6 & : & 18 & : & 27 \\ 8 & : & 24 & : & 36 \end{array}$$

$$\begin{array}{ccc} R & : & W \\ 1 & : & 4 \end{array}$$

Does not work...

ok it is a multiple of 4
Not a mult of 4.

Andy, Bob, and Cindy work in a company. The ratio of Andy's age to Bob's age is 4:3 and Bob's age to Cindy's age is 4:3. If the sum of their ages is 111, how old is Bob?

- a) 48
b) 36
c) 32
d) 27
e) 24

$$\begin{array}{ccc} A & : & B & : & C \\ 4 & : & 3 & : & 3 \\ & & 4 & : & 3 \end{array}$$

$$\begin{array}{ccc} 16 & 12 & 9 \\ 16x + 12x + 9x = 111 \\ 37x = 111 \\ x = 3 \end{array}$$

$$Bob = 12x = 36$$

The ratio of women to children at a certain party is 2 to 5 and the ratio of children to men is 3 to 4. If there are more than 13 and less than 20 women at the party, what is the number of men at the party?

$$\begin{array}{ccc} W & : & C & : & M \\ 2 & & 5 & & \\ & & 3 & & 4 \end{array}$$

$$6 : 15 : 20$$

$$\begin{array}{ccc} \times 3 & & \\ 12 & & \\ 18 & \rightarrow & 60 \\ 24 \dots & & \end{array}$$

Amount of women must be a multiple of 6

83. Michael sells twice as many \$20 tickets as Thomas, and Thomas sells three times as many \$10 tickets as Michael does. If there are only \$10 and \$20 tickets, how many tickets does Michael sell?

(1) Thomas sold a total of 35 tickets.

(2) Together Michael and Thomas sold 70 tickets for a total of \$1,000

	m	:	T	Total	(1)	$x + 3y = 35$
20	$2x$:	$1x$	$3x$	$-(2)$	$3x + 4y = 70$
10	$1y$:	$3y$	$4y$		$3x(20) + 4y(10) = 1000$

(B)

84. An investment fund manager is considering three stocks, P , Q , and R for her portfolio. For the cost of 1 share of P , she could buy 3 shares of Q or 5 shares of R . If she invested an amount equal to the cost of 4,500 shares of Q but purchased equal numbers of shares of P and R and no shares of Q , how many shares of R did she buy?

$$1(P) = 3(Q) = 5(R)$$

$$P : Q : R$$

$$15 : 5 : 3$$

Ratio of the costs

$$\begin{aligned} 3x &= 4y \\ x &: y \\ 4 &: 3 \end{aligned}$$

$$4500(5)$$

Total money spent

$$= 15x + 3x$$

Unknown equal amt of shares

1 share of P costs 15

1 share of Q costs 5

1 share of R costs 3

$$4500 \cdot 5 = 18(x)$$

$$x \cdot 500 \cdot 5 = 2 \cdot x(x)$$

$$1250 = x$$

x = The amt of shares of Q and R

85. Andy and Bob enter into a partnership. Andy invests \$5000. At the end of 3 months, he withdraws \$500 and at the end of 7 months he withdraws \$900. If Bob does not withdraw any money throughout the year and gets \$800 as his share of the total profit of \$1800 at the end of the year, how much did he invest?

STEP 1 Find the weighted average of Andy's investment for the year

$$\begin{array}{r} 5000(3) \\ 4500(4) \\ 3600(5) \end{array} \left. \vphantom{\begin{array}{r} 5000(3) \\ 4500(4) \\ 3600(5) \end{array}} \right\} \begin{array}{r} 15000 \\ 18000 \\ 18000 \\ \hline 51000 \end{array}$$

$$\frac{51000}{12} = 4250$$

$$\begin{array}{r} 4250 \cdot 12 \cdot 12 \\ \hline 12 \\ 3 \\ 25 \\ 17 \\ \hline 175 \\ 250 \\ \hline 4250 \end{array}$$

STEP 2 Ratio

$$\frac{A}{B} = \frac{X}{800}$$

$$\frac{4250}{1000} = \frac{X}{800}$$

$$\frac{4250 \cdot 800}{1000} = X$$

$$3400$$

$$\frac{\left(\frac{51000}{12} \right)}{1000} = \frac{X}{800}$$

$$\frac{17 \cdot 200}{51 \cdot 800} = X$$

$$3400 = X$$

Ratio
✓