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8 Friends: Tom, Jane, Sue, Mary, Linda, Craig, Bob and Andy, meet to play a game. After talking for a little while they decided they should get something to eat before they play. One person is selected at random to get some pizza and another is selected to get something to drink.

1. How many ways can two people be selected to get the food and drinks?

$\binom{8}{2}$

P D

$8 \cdot 7 = 56$

In this case we want the groups and the permutations in each group

Group SUE Tom $\neq$ Tom Sue

Tom gets drinks Sue gets pizza

or

Sue gets drinks Tom gets pizza

2. At the pizza parlor there is a special on 3 topping pizzas. If there are 12 different toppings to choose from how many different pizzas could be created?

$$\binom{12}{3} \qquad \frac{12 \cdot 11 \cdot 10}{3!} = 220$$

In this case we want only the groups.

Getting mushroom, onion and olives is the same no matter which order you select them.

3. At the beverage shop there are 6 different drinks to choose from. If two are chosen, how many groups of 2 different kinds of drinks can be selected?

$\binom{6}{2}$ $\frac{6 \cdot 5}{2!} = 15$

In this case we want only the groups.

Getting coke and sprite is the same as getting Sprite and Coke.

4. Back at the party the pizza and the drinks are put in the table. How many different combinations of two 3 topping pizzas and two different dinks could have been chosen?

$$\binom{12}{3} \cdot \binom{12}{3} \cdot \binom{6}{2}$$

$$\frac{12 \cdot 11 \cdot 10}{3!} \cdot \frac{12 \cdot 11 \cdot 10}{3!} \cdot \frac{6 \cdot 5}{2!}$$

$$220 \cdot 220 \cdot 15$$

$$\begin{array}{r} 22 \\ 22 \\ \hline 44 \\ 44 \\ \hline 484 \\ 15 \\ \hline 2420 \\ 4840 \\ \hline \end{array}$$

$$\boxed{726000}$$

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5. Tom is unhappy with the pizza selection and goes back to get a pizza that he would like. If on his pizza he won't eat mushrooms and has to have sausage as one of the toppings, how many different pizzas could he select?

$$\binom{10}{2} \qquad \frac{10 \cdot 9}{2!} = 45$$

Since mushrooms is no longer an option there are only 11 toppings left.

AND since he has to have sausage on his pizza he is really only selection 2 toppings from the ten remaining toppings.

In this case he is NOT selecting 3 toppings from 11 because the sausage is a must. The other two toppings are completly open so therefore he is selecting 2 from 10.

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After lunch they begin to play some games.

6. In one game they need one group of four people. From the four girls and four boys how many different groups of two boys and two girls can be created?

4 boys   4 girls

BB   GG

$$\binom{4}{2} \cdot \binom{4}{2} \qquad \frac{4 \cdot 3}{2!} \cdot \frac{4 \cdot 3}{2!} = 36$$

In this case we are taking two different groups and putting them together to make one larger group. In such cases solve for each group separately and then multiply them together.

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7. How many different groups of three girls and 1 boy can be created?

4 boys 4 girls

b ggg

$$\binom{4}{1} \cdot \binom{4}{3}$$

$$4 \cdot \frac{4 \cdot 3 \cdot 2}{3!} = 16$$

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8. How many different groups of four can be created that have at least one boy?

4 boys 4 girls

B B B B

B B B G

B B G G

B G G G

G G G G

any group of 4 — all girls

$$\binom{8}{4} - \binom{4}{4}$$

$$\frac{\cancel{8} \cdot 7 \cdot \cancel{6} \cdot 5}{\cancel{4!}} - \frac{4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1}$$

$$70 - 1 = 69$$

In such cases when you are asked for at least one: Simply calculate the whole minus what is not desired.

9. In the next game six people are needed. How many ways can the eight friends be divided into two teams of three people?

$$\frac{\frac{8\cdot 7\cdot 6}{3!}\cdot\frac{5\cdot 4\cdot 3}{3!}}{2!} \qquad \frac{56\cdot 10}{2!} = 280$$

Not to worry about this case too much; it is very uncommon on the test.

The reaseon why you need to divide the whole thing by 2! in the end is to remove the duplication.

Team A playing Team B is the same as Team B playing A...

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Next the eight friends play a game in which they need to stand in a straight line.
10. How many different ways can they all stand?

$$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 8!$$

11. How many different ways can they all stand if the four boys and the four girls won't stand next to each other?

A boy will not stand next to a boy and a girl will not stand next to a girl.

$$\underbrace{\overbrace{\underline{B}\quad \underline{G}\quad \underline{B}\quad \underline{G}\quad \underline{B}\quad \underline{G}\quad \underline{B}\quad \underline{G}}^{4!}}_{4!} \qquad 4!\cdot 4!$$

or

$$G\quad B\quad G\quad B\quad G\quad B\quad G\quad B \qquad 4!\cdot 4!$$

$$\begin{array}{r} 24 \\ \times\ 24 \\ \hline 96 \\ 480 \\ \hline 576 \end{array}$$

$$4!\cdot 4! + 4!\cdot 4!$$

$$24\cdot 24 + 24\cdot 24$$

$$576 + 576$$

$$\begin{array}{r} 576 \\ \times\ 2 \\ \hline \boxed{1152} \end{array}$$

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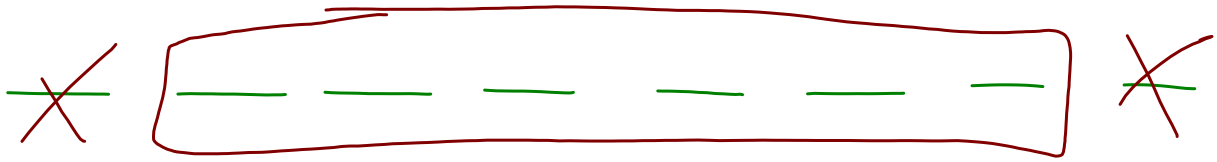
12. How many different ways can Sue and Mary stand?

m ___ ___ ___ ___ ___ ___ ___ ___

s s s s s s s

$8 \cdot 7 = 56$

13. How many different ways can Sue and Linda stand if they won't stand on either end?



$\binom{6}{2}$ $6 \cdot 5 = 30$

14. How many different ways can Bob and Andy stand if they won't stand next to each other?

___ ___ ___ ___ ___ ___ ___ ___

all ways

$$\binom{8}{2} - 7 \cdot 2!$$

$$8 \cdot 7 - 14 =$$

$$56 - 14 = 42$$

15. How many different ways can Bob, Andy and Sue stand if they won't stand next to each other?

any three

not wanted

$$\binom{8}{3} - 6 \cdot 3!$$

$$\boxed{8 \cdot 7 \cdot 6 - 6 \cdot 3!}$$

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16. How many different ways can they all stand if Tom will not stand on either end?

$6 \cdot 7!$

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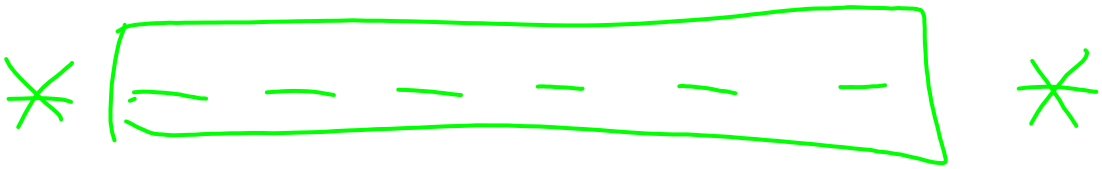
17. How many different ways can they all stand if Tom and Jane will not stand on either end?



$6 \cdot 5 \cdot 6!$

18. How many different ways can they all stand if Tom and Jane must stand together, Sue Mary and Linda must stand together and Craig Bob and Andy must stand together?

$\boxed{\text{Tom Jane}}$ $\boxed{\text{Sue Mary Linda}}$ $\boxed{\text{Craig Bob Andy}}$

$2! \cdot 3! \cdot 3! \cdot 3!$

each group can move

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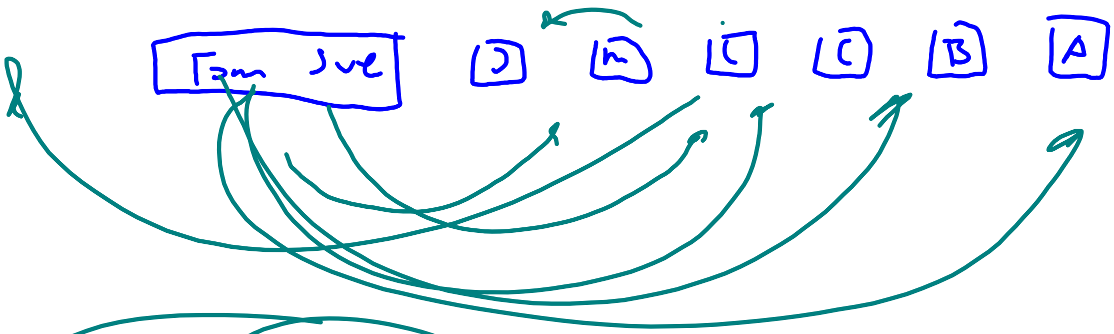
19. How many different ways can they all stand if only Tom and Sue must stand together?



$7! \cdot 2!$